

Further to Norman Neidell's series....

Holistic migration

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In principle, a hologram and a seismic section are the same. When a hologram is illuminated or a seismic section is migrated, each depicts the entire image of the unattainable object. Moreover, any part of the hologram also depicts the entire image of the object. The same is true in the seismic case if conventional migration is replaced by holistic migration. Holistic migration is a process in which any part of the seismic section produces the entire image of the subsurface. Holistic methods allow wavefield images to achieve resolution beyond that predicted by conventional digital processing techniques. Suppose that a given number of shotpoints and detectors yields a certain resolution of the subsurface structure under conventional migration. Then only one-half or one-fourth as many shotpoints and detectors can give the same resolution under holistic migration. The image might not be quite as clear, but the same fine structure will be present. The implication is that in a seismic survey the number of shotpoints and detectors can be greatly reduced without adversely affecting the results. Because the cost of a seismic survey depends directly upon the number of shotpoints and detectors used, the holistic method can reduce the cost of a 3-D seismic survey greatly, often to one-fourth the previous cost or less. The fine details in geologic structure that fall through the cracks in conventional processing are captured in holistic processing.

Norman Neidell has given new approaches for use in seismic acquisition and processing. His novel views on spatial sampling challenge concepts that have traditionally been taken for granted. According to Neidell, the Nyquist theorem is not as restrictive as is commonly applied. Neidell's approach has generated much interest as well as some controversy. Neidell has asked me to give a theoretical justification for the new approach.

Seismology and holography. The point of this paper is to establish the principle that a seismogram is a hologram. The understanding of this principle opens up new vistas in seismic exploration, and in particular justifies important new acquisition and processing methods. Neidell's methods are now the property of Zydeco

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Energy Inc. of Houston, and they can be obtained from that source.

The hologram was invented and named by Dennis Gabor in 1948. In simple terms, holography represents a method of recording an interference pattern on a plane (the hologram), and thus from the hologram the entire picture in 3-D space can be obtained. The word is Greek for whole (*holo*) message (*gram*). The hologram contains the required information to reconstruct a 3-D picture of the object.

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Limits of resolution

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In the early days of the oil-industry use of the seismic method, it seemed natural to pick reflection events from paper records and to migrate them using ray tracing. With the advent of computers there was an increase in sophistication, and then it seemed natural to keep in mind that the observed phenomena could be described by the use of partial differential equations. Digital recording obviously yielded only discrete samples of the generated wavefield, whereas the differential equations assumed continuous observations.

It thus seemed obvious that the way to proceed was to use the digital samples to construct a corresponding continuous wavefield. Because the actual work was done using digital computers, discrete mathematics was used in the final analysis.

Now it is well known that with N observations no more than N parameters can be determined. It seems not to be equally appreciated that in this paradigm the N observations are being used to construct an N parameter interpolation, namely the band-limited functions corresponding to the sampling interval. By using various forms of finite-difference equations, this wavefield is continued down, and the time coincidence of this upgoing and the downgoing wave from the source is used to identify a reflection and its corresponding amplitude. Since we are assuming that we have reconstructed the continuous signal, we are free to interpolate it to a finer grid, and thus apparently locate the reflected event more precisely. The conventional migration algorithms can then yield apparent higher resolution. There is plenty of experience to indicate that it can be more than apparent, because the number of samples per unit of space is not uniform in an inhomogeneous medium. However, using a finer grid increases the memory and time demands on the computation and is thus not routinely justified. This suggests that the same result should be capable of being accomplished more economically.

It is important to note that the Nyquist limit here refers to the wavenumber, and not to frequency. The Nyquist limit on frequency is inviolate. Even though not relevant here, it can be noted that if the fundamental frequency band is empty, the Nyquist limit legitimates the use of frequencies in whichever higher band is not

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(Eisner, continued from p. 313)

empty. Whereas frequencies are conserved, wavenumbers are not necessarily conserved. They are conserved in a uniform medium, but in an inhomogeneous medium refraction leads to changes in wavenumber as the wave travels. Thus we may have regions of increased resolution where rays converge, at the expense of decreased resolution where rays diverge. Resolution depends on the wavenumber at the target rather than on some "average" wavenumber. Many discussions on resolution lose sight of this point and equate this average resolution with resolution at specific points. While it is true that regions of improved resolution exist at the expense of other regions of decreased resolution, interpretation of the data often concentrates on the local situation. The "average" may be more relevant in advance planning of the fieldwork, but even if not predictable in advance, there is no reason not to exploit improved resolution when the opportunity presents itself. Since velocity usually increases with depth, correspondingly increasing wavelengths, resolution generally decreases with depth. A velocity lens could magnify some particular area in the subsurface and give particularly high resolution on a very local scale.

In this standard approach, no use is made of the fact that the hardware collects the samples of the arriving wave by the use of a sample window which is about $1/40$ of a millisecond. If there is useful arriving energy at the corresponding frequency, it would be possible, using current technology, to record up to that frequency. This could be done by a collection of digitizers whose sample windows were staggered so that some sample window always remains open. It seems that this scheme is not economically justified, although possible. Currently we lose higher frequencies and wavenumbers by the use of antialiasing filters and arrays. We do not know how much useful information we are thus discarding, since our subsequent processing is unable to use it. Since there is no sharp, definite upper limit on the frequencies which can be transmitted through the earth, in the final analysis resolution is limited by the signal-to-noise ratio. This ratio can be improved by stacking repetitions, but because it only grows with the square root of the number of repetitions, in practice there is a limit, but it is not definite. Using a single frequency, with sufficient signal-to-noise ratio by using the phase, a single diffractor can be located to arbitrary accuracy. However, when more than one diffractor is at issue, separating the arrivals from the different diffractors complicates matters. In estimating the Nyquist limit frequency, there is often the implicit assumption that loss of high frequencies is due to attenuation. However, it is often observed that later arrivals contain higher frequencies than earlier arrivals, which demonstrates that caution is needed here.

(Robinson, continued from p. 313)

Reflection seismology is a geophysical method of mapping the subsurface structure of the earth from knowledge of events reflected from the subsurface layers. The earth's sedimentary layers are approximately horizontal, but they do have features such as anticlines, unconformities, and faults that can serve as traps for petroleum. The received seismic signals (called traces) recorded at the surface make up the seismic section. The traces give the reflected events as a function of time. In order to map the subsurface, the geophysicist must

With regard to antialiasing filters, they can equivalently be located in the source pulse itself, in analog filters preceding the digitization, or subsequent to the digitization by discrete filters.

With regard to arrays and wavenumber filtering, there seems to be some confusion. As long as an array simply provides a weighted sum of the constituent traces, it makes no difference whether a filter is applied separately to each trace before combining, to the sum after they are combined, or to the source pulse itself. The wavenumbers present in the source pulse do not determine those at the detector, because if the velocity at the detector is low enough, arbitrarily high wavenumbers may be present there. If the array applies different filters to each trace before summing, the effect may be different for waves arriving from opposing directions. For many areas with surface problems, this needs to be appreciated and would tend to favor nonuse of arrays.

Since increased resolution is often desired, it is natural to inquire if alternative schemes for interpretation can be devised which would allow the use of this information, when it is present.

Starting with a detailed model of the exploration target, one can calculate the response to any desired accuracy, given time and resources. Then one can compare the calculated responses which correspond to the observed responses and recursively improve the model. To date this approach has not been successful, primarily because the measures used to evaluate how well the model fits the observed data have too many localized minima. It has the virtue that it can deal with irregularly spaced data and can use the actual time accuracy of the data. It thus suggests trying to find schemes which incorporate these virtues but otherwise resemble those in current use.

One possible scheme is to represent a field record as a linear superposition of the responses to a set of virtual images. These could be virtual images of the pointlike source or of point diffractors. 3-D records would provide polarization information which would identify the wave type. This analysis could be done with no preliminary model of the earth. Representing the traces in terms of wavelets might provide a useful entrée.

The previous paragraphs suggest that current algorithms and field procedures may be capable of improvements yielding higher resolution. Neidell presented the results obtained with an algorithm which represents the earth model as a set of point diffractors, which supports this conclusion. The use of the conventional approach to obtain the big picture followed by methods which do not include the usual constraints seems to be a promising approach for future development. **E**

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convert the seismic section into a spatial image of the subsurface. This computed image is called the migrated section. Whereas the seismic section is a time section (time signals plotted against horizontal coordinates), the migrated section is a depth section (depth functions plotted against horizontal coordinates). The many data-processing methods falling under the general title of migration represent various ways of creating the depth section.

Migration is a manifestation of the general problem of determining the internal structure existing in a medi-

um when access is confined to a surface on the boundary of the medium. This problem is common to many applications of wave motion for sensory purposes, as in radar, sonar, and ultrasonics, as well as in exploration seismology.

Migration transforms the seismic section (a time section) into the migrated section (a depth section). Migration starts with the recorded wavefield, that is, the wavefield incident upon a measuring array placed upon the surface of the earth. In one form or another, migration reconstructs the wavefield at any point within the earth by use of various approximations to the wave equation and/or the associated eikonal equation. The reconstructed wavefield is then imaged to produce the migrated section. This approach is well suited to computer calculation. It is important that a distinction be made between wavefield reconstruction and imaging. Wavefield reconstruction is the determination of the wavefield over a region of interest. Imaging involves the making of a picture of the geometrical distribution of the reflecting surfaces within the medium. The input to the migration process is the seismic section, that is, the totality of the recorded signals received at the surface of the earth. The first step of the migration process is wavefield reconstruction. The second step is the imaging of the wavefield to yield the required image (called the migrated section) as the output.

The objective of this paper is to show that, in principle, a seismic section is a hologram, and that the migration process corresponds to the illumination of the hologram in order to produce the required image. A hologram is made by shining laser light at an object. Half of the laser beam never hits the object because it is reflected from a mirror placed in its path. This reflected light, called the reference beam, is directed to a photographic plate. The other half of the laser beam finds the object. Each point P of the object acts as a diffraction point and spreads light in all directions. The diffracted light from point P reaches every point on the undeveloped plate. When the two halves of the laser light meet at the plate, they interfere with each other. The resulting interference pattern is recorded on the plate (Figure 1).

When developed, the plate becomes the hologram. When a laser beam is later directed through the hologram (with the object removed), a virtual image of the object unfolds from the wave pattern and projects three-dimensionally in space. If a viewer walks around this chimerical object, he sees its image from different perspectives as he would see the real object. The reason is that the whole object has been recorded at every point of the interference pattern on the hologram. A hologram is a 2-D photographic plate that allows us to see a faithful reproduction of a scene in three dimensions (Figure 2).

Cutting a piece from the hologram and sending the laser beam through the fragment also produces an image of the whole object, although this image may not be quite as sharp. Let us now discuss why every piece of a hologram contains the entire picture. Each diffraction point of the object sends out waves that reach every point on the hologram. As a result each point on the hologram contains a contribution from every point on the object. It follows that every point on the hologram contains the entire picture. Thus when either the entire hologram is illuminated or just a section of it, we see the image of the entire object. Of course the more points

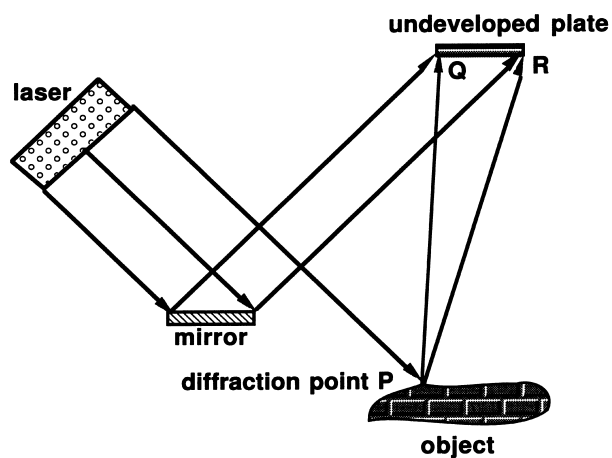


Figure 1. Making a hologram. The light rays diffracted from point P on the object are directed to all points on the undeveloped plate, such as points Q and R.

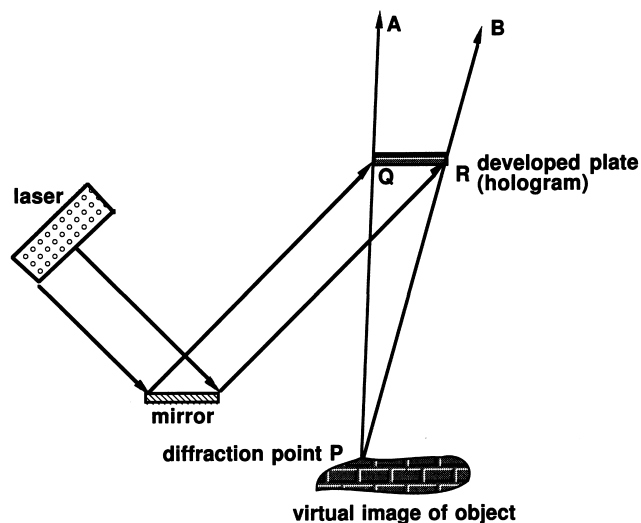


Figure 2. Illuminating a hologram. The developed hologram bends the laser rays so as to form a virtual image of the object. The actual object has been removed beforehand. In looking at the hologram we see the virtual image at different perspectives from A and B.

used on the hologram, the better the quality of the picture. The characteristic that each part of a hologram contains the entire picture has widespread implications in seismology.

In an ordinary camera, a lens is used to form an image of the object on the plane of the photograph film. Light reflected from a given point on the object is directed by the lens to the corresponding point on the film. Thus there is a one-to-one relationship between points on the object plane and points on the photograph film. Moreover, all the light that reaches the film comes from the object. There is no secondary source as in the case of a hologram in which half of the laser light (the reference beam) is reflected from the mirror and does not reach the object. Let us now compare the ordinary camera with holography. In making a hologram, no image-forming lens is used; each point on the object diffracts light to

every point on the hologram plate. In other words, there is a one-to-many relationship between points on the object and the points on the hologram plate. Thus every part of the hologram plate is exposed to light diffracted from every part of the object. In addition, the total light that reaches the hologram plate is made up of two parts, namely the part of the beam (the reference beam) that is reflected from the mirror and the part of the beam that is used to illuminate the object. These two parts produce the interference pattern recorded on the hologram. That is, one set of wavefronts is from the reference beam, and the other set is from the light diffracted from the object. A hologram is a recording of the interference pattern resulting from the combination of these two sets of wavefronts.

Because vast amounts of information are recorded on a hologram, the film used must have a resolving power much greater than ordinary fine-grain photographic film. The developed hologram contains the intensity of the reference beam modulated by waves from the object. If we look at a hologram we see no recognizable image. The hologram is dark where the object wave and the reference wave arrive in phase, and it is light where the object wave and reference wave arrive out of phase. Thus the intensity of the hologram corresponds to a given phase difference between the object waves and the reference waves and is unaffected by a change in sign of that difference. A hologram is a photograph of microscopic interference fringes and appears as a hodgepodge of whirly lines. When the hologram is placed in a beam of laser light (with the object gone), the light rays are bent by the hologram to produce rays identical to the original rays diffracted by the object. When viewed by the eye, the bent (or diffracted) rays produce the same effect as the original diffracted rays. When we look through the hologram we see a full realistic 3-D virtual image as if we were viewing the object through a window. When we move our eyes and look down the sides of the object and when we lower our eyes and look underneath the object parallax is evident as in real life. The entire wavefield on our side of the hologram has been reconstructed by the illumination of the hologram by the laser light. We see the object as a virtual image, even though the original object is no longer present.

The seismic section as a hologram. Consider now the problem of seismic exploration for petroleum. The way of carrying out a seismic survey is, of course, different from the way a hologram is made. In holography, electromagnetic (light) waves are used. In the seismic case, acoustic (sound) waves are used. The laser waves are extremely narrow band, and are, in fact, almost pure sine waves. In contrast, seismic waves are broad-band pulse-type waves. An important feature of seismic exploration is that there is an absolute reference time (the time of the shot); this feature makes it unnecessary to use a reference beam as in holography. On the photographic plate making up the hologram, only the intensity pattern of the interference is recorded.

In seismic exploration, the whole interference pattern is recorded as time-varying signals (the seismic traces). In holography, the exterior of an opaque object is imaged. In seismology, the entire interior structure of the earth is imaged because solid rock is transparent to seismic waves. Seismic surveys are taken on the surface of the earth in order to determine the underlying 3-D structure. The sources and receivers are at grid points on the surface of the earth and correspond to the holographic plate. The reflecting horizons are the interfaces between the sedimentary rock layers within the earth. These reflecting horizons represent the geologic object to be imaged. Waves reflected from the subsurface structure are returned to the receiving instruments on the surface and recorded as seismic traces. The collection of all the traces is called the seismic section. The seismic section is the counterpart of the developed holographic plate. In fact, a seismic section is a hologram in the true meaning of the word.

In seismic exploration, the subsurface object (the geology) is made up of a collection of reflection surfaces. However, any surface can be approximated by a set of closely spaced points lying on the surface. Each point of the set acts as a point diffractor. That is, a reflector is nothing more than a continuous collection of diffraction points. Rays from a diffraction point reach all points on the earth's surface. The seismic traveltime curve for the rays from a point diffractor is called the diffraction curve. The diffraction curve is the seismic section (or hologram) for the point diffractor (Figure 3).

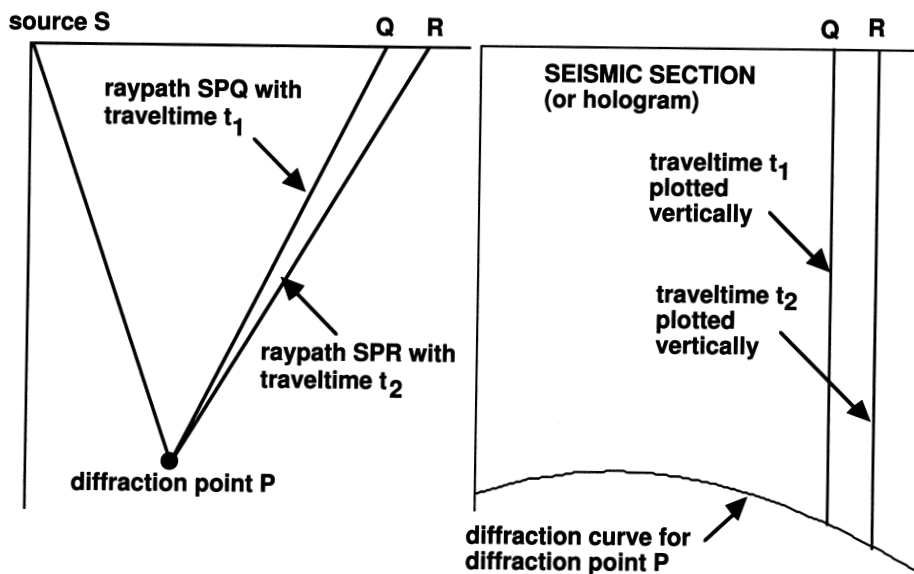


Figure 3. Recording a seismic section (or hologram). The returned signal from a diffraction point gives a hyperbolic-shaped diffraction curve. The geologic structure can be considered as composed of many diffraction points. The resulting seismic section is the superposition of all the diffraction curves.

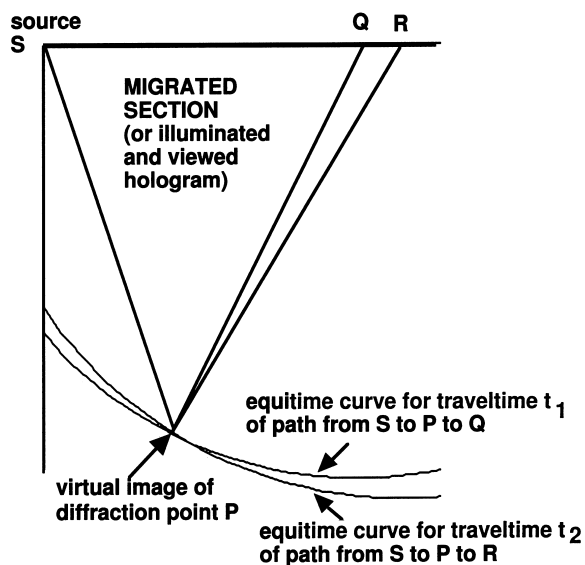


Figure 4. A representation of the process of migration. For each of traces Q and R, the amplitude value for the given traveltime is mapped into the subsurface along the entire equitime curve, that is, the curve formed by the loci of points for which the traveltime from source to diffraction point and back to receiver is constant.

When the layers of sedimentary rock lying beneath the earth's surface are relatively flat and unexceptional, the seismic section can give a remarkably direct indication of the structural configuration. Because the depth that a wave penetrates and its traveltime are related by the velocity of the traveling seismic waves, there is a correspondence between the depth axis and the time axis. Thus the recorded seismic data (the seismic section) as a function of horizontal coordinate and time give a rough picture of the cross-section of the earth that is a function of horizontal coordinate and depth. In this sense the seismic section is like a picture, albeit distorted, of the subsurface geologic structure. In the same way, a hologram is a picture of the image, but it is so distorted that no sense can be made of it as such.

In the case of a complex subsurface structure, a seismic section can be misleading and hard to interpret geologically even when record quality (such as the continuity of reflected events and the ratio of reflected signal to background noise) is excellent. When the underground structure shows large deviations from horizontal layering, the subsurface position of the reflecting point on a reflecting interface does not lie under the shot-receiver midpoint but is displaced to one side or the other of the midpoint. The direction and magnitude of this displacement depend on the direction and magnitude of the dip of the reflecting interface. Therefore, the seismic section must be processed so that the event is moved (or migrated) to a position representing the correct spatial coordinates of the reflecting horizon. This processing technique, known as migration in the geophysical industry, is necessary in order to delineate correctly the oil-bearing structural traps (folds, faults, and domes). Migration is the seismic digital processing method that corresponds to the optical processing that takes place when the developed hologram is illuminated and viewed in order to yield the virtual image. In the true sense of the word, migration is the seismic coun-

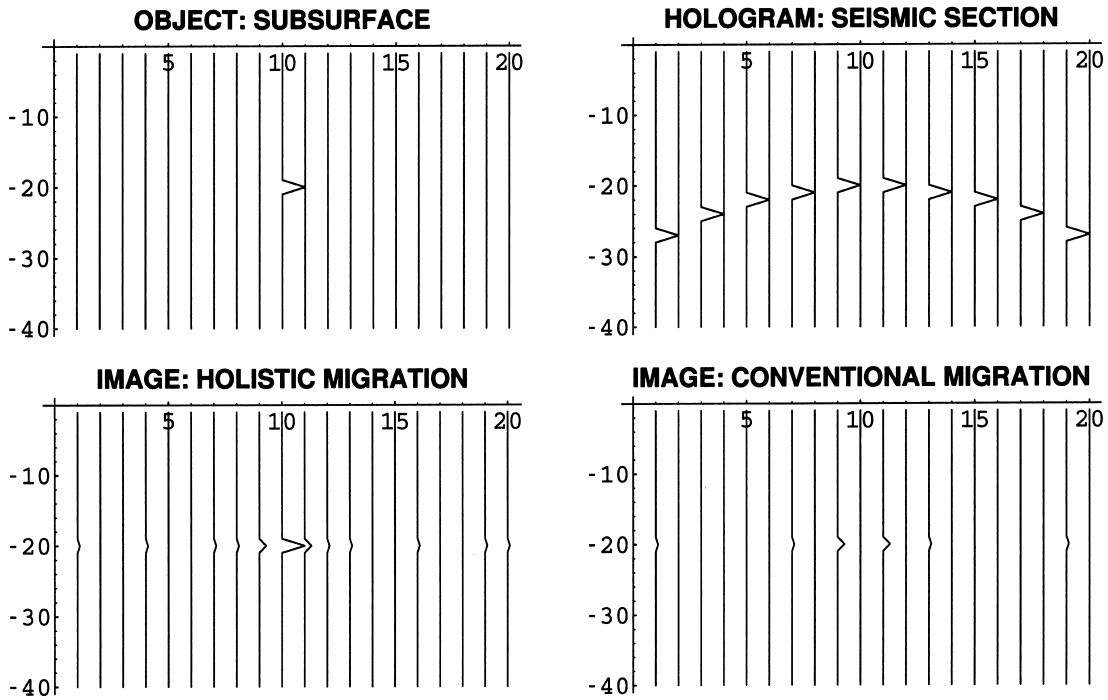


Figure 5. The object is a point diffractor. Two methods of migration are applied to the seismic section. Only holistic migration is successful in detecting the point diffractor.

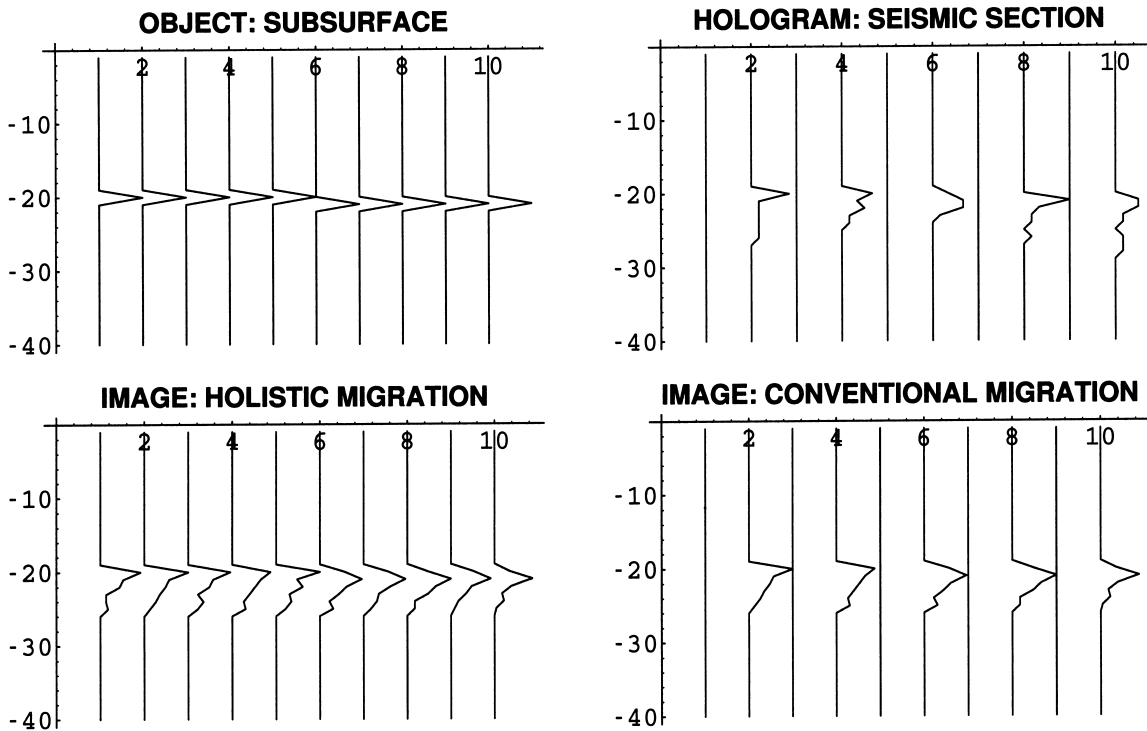


Figure 6. The object is a sharp diffracting edge. Two methods of migration are applied to the seismic section. Only holistic migration is successful in detecting the location of the fault.

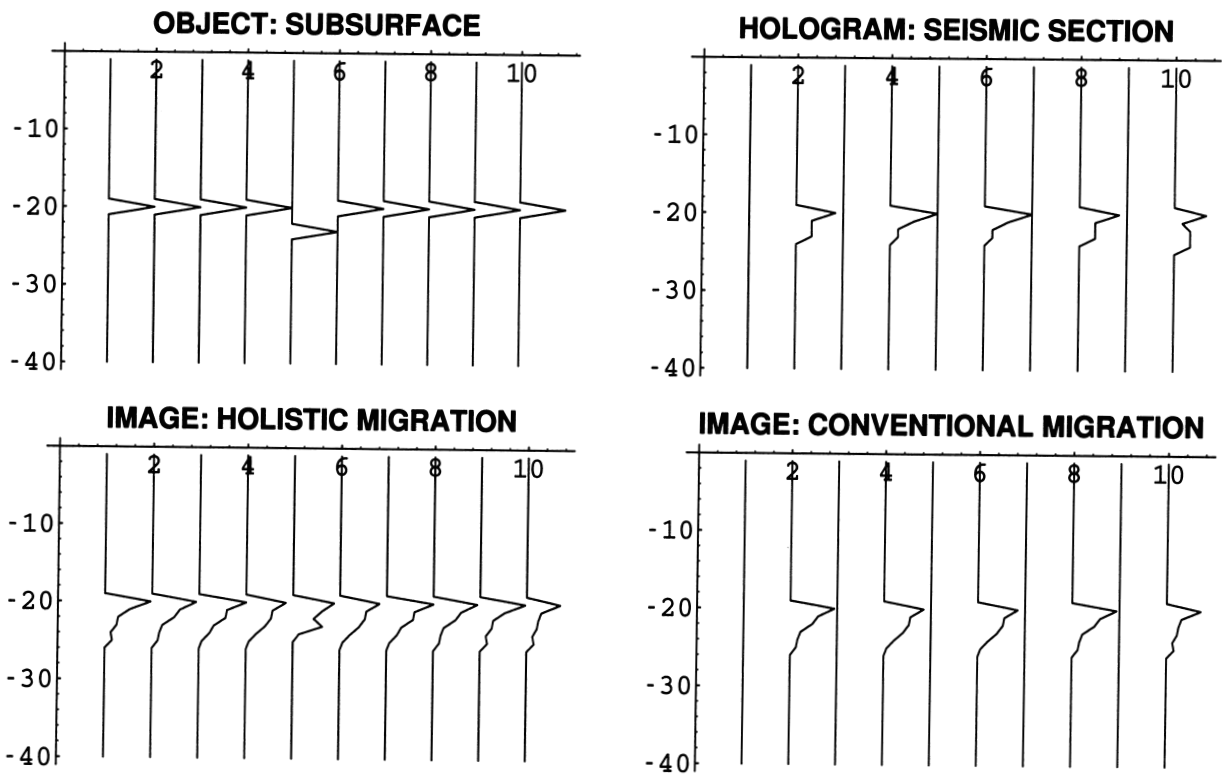


Figure 7. The object is a small flaw on an interface. Two methods of migration are applied to the seismic section. Only holistic migration is successful in detecting the flaw.

terpart of illuminating and viewing the developed hologram to bring out the virtual image in the eye of the observer. In the seismic case, the virtual image, or migrated section, is a 3-D reconstruction in the computer of the geologic objects that can be as much as five miles below the surface of the earth.

Migration as illumination. The migration of seismic data is performed routinely in seismic processing regimes. Various techniques for migration are commonly available. The basic theory of migration can be explained in terms of diffraction curves and equitime curves. For purposes of exposition, it is easier to consider the case where the seismic data are taken along a single surface line, and to regard the underlying earth structure as 2-D. The two spatial dimensions are the horizontal coordinate x and the depth coordinate z . The variable x can take on any positive or negative value, but the variable z must be positive. Also, time t must be positive. Because z and t must be positive, there is a certain duality between them.

There is nothing in our exposition that cannot be extended to three spatial dimensions; the extension is straightforward and involves no new principles.

The recorded seismic section forms the starting point for migration. The seismic section is produced by a seismic survey in which each source and receiver pair produce a seismic trace. The totality of traces make up the seismic section. The wave equation describes the motion of the waves generated by the source. However, the seismic section does not correspond to a wavefield resulting from any single experiment because the

sources are excited sequentially, not simultaneously. A seismic trace records the two-way traveltimes from a given surface source to all possible diffractors and then back to a given surface receiver. The seismic section contains the traces for all the source-receiver pairs used in the survey.

In seismic work, the sources and receivers are at the surface of the earth and reflectors are at depth. A certain type of idealization deserves special study, namely, the case of a point reflector, which is also called a point diffractor. When such a diffraction point is illuminated by a surface source, the diffraction point acts as a secondary source and hence sets off outgoing wave motion in all directions. Any reflecting surface may be considered as being made up of a dense set of point diffractors.

The time coordinate of each event on a trace gives the two-way traveltime from the given source to an unknown diffraction point and back to the given receiver. Although the position of the diffraction point is unknown, it is known that it must lie somewhere on the curve defined by the given source, the given receiver, and the given two-way traveltime. This curve formed by the totality of diffraction points, all of which have the same traveltime for the given source/receiver pair, is called the equitime curve. An equitime curve is defined by the source and receiver positions and the traveltime of an event on the seismic trace recorded for that source/receiver pair. In other words, a equitime curve gives the locus of all possible diffraction points associated with a given source, receiver, and traveltime of an event. The equitime curve for each event on a seismic trace can be computed, and the actual diffraction

point that produced that event must lie a some point on that curve.

The migration problem can now be stated in the following terms. The equitime curves can be computed from the known data. The problem then is to determine where on that curve the true (geologic) diffraction point lies. The general idea of migration can be described as follows. Take an event on a trace in the seismic section and throw it out onto its equitime curve in the spatial x and z dimensions. Keep repeating this process so as to obtain the totality of the equitime curves from each data point of every trace in the seismic section. Place all of these curves on the same graph. By the linear superposition principle, the result is the migrated section. That is, the migrated section is the superposition of all the equitime curves.

In the case of constant velocity, equitime curves are elliptical with the source and receiver as the foci. Generally, equitime curves are more complicated and must be computed according to a velocity function. A useful subsurface image is produced because of the constructive and destructive interference among the equitime curves. For example, equitime curves from neighboring traces will all intersect at a true reflection point, adding constructively to produce an image of the reflector in the form of a high-amplitude output. For a continuous true reflecting surface, equitime curves from adjacent traces are tangent to this surface and produce an image of the reflecting surface by constructive interference of overlapping portions of adjacent equitime curves. On the other hand, in subsurface regions without reflecting

bodies, the equitime curves tend to cancel because of random interference effects.

In summary, migration takes the value of the trace at a given time and places this value evenly along the equitime curve defined by the given time. The migrated section is the sum of all the values on the equitime curves (Figure 4).

Holistic migration. The migration methods described to this point represent the existing way that migration is done (conventional migration). The use of the holographic principle in migration defines a new method of migration, appropriately called holistic migration. Conventional migration fails to recognize that a seismic section is a hologram. A portion of a hologram can produce the same picture as the entire hologram. Suppose, for cost reasons, a 2-D survey must be laid out with detector spacing equal to two spatial units. It is a common assumption that this spacing of two units governs the Nyquist frequency, so the processing is done at a spacing of two units.

Accordingly, by use of conventional migration, the resolution at depth is forced to be two spatial units. If it is desired that the resolution at depth be one spatial unit, then the common belief is that detectors must be laid out at a spacing of one spatial unit. The cost of a 2-D survey with one-unit spacing is about twice the cost of the survey with two-unit spacing. Thus, according to commonly accepted beliefs, to double precision, one must double cost, or thereabouts. This situation need not be so. One can double precision without any appreciable increase in cost. Alternatively, one can obtain the same precision at the half the cost. The way is to use holistic migration in seismic processing instead of conventional migration.

Observe that the seismic section from a two-unit sampling is a subset of the seismic section with one-unit sampling. That is, the two-unit section can be obtained from the one-unit section by taking every other trace. In other words, the two-unit section (piece of the hologram) is obtained by cutting a piece from the one-unit section (the hologram). Because the piece of a hologram gives the same image (in all its detail) as the hologram, the same detailed subsurface image is produced by the seismic section with two-unit sampling as by the more costly seismic section with one-unit sampling.

Three examples comparing conventional migration with holistic migration follow. In each example the same seismic section is used for both conventional migration and holistic migration. The first example is the case of a point diffractor as shown in Figure 5. Diffractions differ from true reflections in that the energy from a shot returns from or near the point diffractor without appearing to obey the reflection law (incidence angle equals reflection angle). Any reflector may be considered as made up of a continuum of diffraction points lying on locus of the reflector. In the example shown in the figure, the given point diffractor falls through the cracks of conventional migration but is easily found by holistic migration.

The second example is that of a sharp diffracting edge at a fault (Figure 6). The final example is the case of a small flaw on an interface (Figure 7). ■

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